COMPUTATION OF LAMINAR FLOW IN MIXED TURBULENT CONVECTION AND ITS INFLUENCE ON HEAT TRANSFER

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The author has derived a total flow energy equation for conditions of turbulent mixed convection on a vertical surface. He proposes correlations describing the development of laminar flow based on systematic solutions of this equation, and has derived a formula for the wall heat transfer.

INTRODUCTION

Many experimenters have noted reduced turbulent fluctuations when natural convection has been added to forced convection. It is well known that this phenomenon is accompanied by reduced wall heat transfer. Unfortunately, the data on the development of this process is basically fragmentary. There are no quantitative evaluations that would predict the development of laminarization. The heat transfer formulas are in most cases empirical and correlate to a specific series of experiments. This paper will attempt to fill the gap.

Qualitative Analysis and Main Equations. As is known, so-called laminarization of the flow results when convection, i.e., the level of turbulent fluctuations falls, and the energy of turbulence falls, which in turn causes reduced heat transfer.

The author proposes the following qualitative explanation of this phenomenon. The addition of secondary Archimedes forces to the forced turbulent flow leads to distortion of the velocity profile and to the flow being pressed more strongly to the wall, and to a corresponding increase of friction. The increased friction requires increased energy expenditure, and therefore the total flow of kinetic energy, made up of the kinetic energy of the mean flow and that of the turbulence, begins to redistribute, increasing the mean flow energy due to a decrease of the turbulence energy. The result is that laminarization begins. At the same time the total flow energy increases due to the Archimedes forces. When the gain obtained exceeds the loss from the friction increase, the flow again begins to turn turbulent.

The redistribution of flow energy in this process occurs as follows. Clearly bending of the velocity profile leads not only to an increase of the mean flow derivative at the wall, but also to reduced velocity in the outer part of the boundary layer. As a result the maximum values of the $\overline{u'v'}$ correlation, fall into a region with lower values of the derivative, which leads to reduced generation of turbulent fluctuations, and to a corresponding reduction of the turbulent friction. The reduction of the friction forces is equivalent to an increase of the kinetic energy of the mean flow.

To describe this process we can use the total energy flux balance equation in the turbulent boundary layer, a special case of which Hinze [1] obtained with no Archimedes forces and no wall influence. Using the transfer equation for the kinetic energy of turbulence obtained in [2] the author has derived an analogous balance equation accounting for wall influence and the action of buoyance forces on the flow. It takes the following form:

$$\frac{1}{2}\frac{d\delta_e}{dx}-\frac{d}{dx}\int_0^\infty \frac{kU}{U_e^3}dy=-\frac{\beta g}{U_e^3}\int_0^\infty U(T-T_e)\,dy+2{,}1\frac{\beta g}{U_e^3}\int_0^\infty \overline{v't'}dy+\frac{\nu}{U_e^3}\int_0^\infty\left(\frac{\partial U}{\partial y}\right)^2dy+$$
$$+\frac{1}{U_e^3}\int_0^\infty \varepsilon dy+\frac{\nu}{U_e^3}\frac{\partial k}{\partial y}\bigg|_{y=0}+\frac{2\nu}{U_e^3}\int_0^\infty\left(\frac{\partial\sqrt{k}}{\partial y}\right)^2dy. \qquad (1)$$

The terms of this equation, from left to right, are: I) the variation of the kinetic energy of the mean flow; II) the variation of the kinetic energy of turbulence; III) the energy

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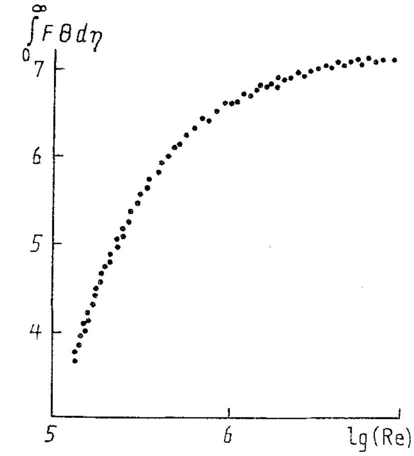

Fig. 1. Computation of $\int_0^\infty F\Theta d\eta$ as a function of Re.

increase due to Archimedes forces; IV) the energy increase due to temperature fluctuations; V) the energy increase due to temperature fluctuations; VI) the turbulence energy dissipation; and VII and VIII) the energy decrease due to the wall influence. As was shown by check calculations, term IV is negligibly small compared with term III, and term VII is neglibibly s small compared with term VIII, and therefore the small terms can be dropped without loss of accuracy.

Since the flow laminarization in the mixed convection appears relative to the forced case, we must write an analogous equation for the pure forced case. It will differ from Eq. (1) only in that terms III and IV are absent.

To describe the processes occurring when natural convection is added to forced convection, we derive an equation for the pure forced case from Eq. (1). Neglecting the small terms we can write

$$\frac{d}{dx}\left[\frac{1}{2}(\delta_{e\mathrm{m}}-\delta_{e\mathrm{f}})\right]-\frac{d}{dx}\int_0^\infty\left[\left(\frac{kU}{U_e^3}\right)_\mathrm{m}-\left(\frac{kU}{U_e^3}\right)_\mathrm{f}\right]dy=-\frac{\beta g}{U_e^3}\int_0^\infty U(T-T_e)_\mathrm{m}dy+\frac{\nu}{U_e^3}\int_0^\infty\left[\left(\frac{\partial U}{\partial y}\right)_\mathrm{c}^2-\left(\frac{\partial U}{\partial y}\right)_\mathrm{f}^2\right]dy+\frac{1}{U_e^3}\int_0^\infty(\varepsilon_\mathrm{m}-\varepsilon_\mathrm{f})\,dy+\frac{2\nu}{U_e^3}\int_0^\infty\left(\left[\frac{\partial\sqrt{k}}{\partial y}\right]_\mathrm{m}^2-\left(\frac{\partial\sqrt{k}}{\partial y}\right)_\mathrm{f}^2\right]dy$$

Here the subscript m denotes "relative to mixed convection," and f denotes "relative to forced convection."

Now the terms describe: I) the variation of the kinetic energy of the mean flow when natural convection is added to the forced case; II) the variation of the turbulent energy; III) the increase of total flow energy due to Archimedes forces; IV) the variation of the energy dissipation of the mean flow; V) the variation of the energy dissipation of the turbulence; VI) the variation of the energy decrease due to the wall influence.

To obtain the numerical characteristics of the process, the system of equations describing the transfer process with turbulent mixed convection [2] was solved twice, once for forced convection, and a second time for mixed convection, but with the same initial conditions and small values of Gr* number, so that the flow was in the forced convection region. All the calculations were made for $0.72 \le \mathrm{Pr} \le 10$. From the results we determined and analyzed the terms of Eq. (2).

Computed Results and Summary Relations. Before analyzing the computed results we shall consider the third term of Eq. (2), counting from left to right. It describes the production of total flow energy due to Archimedes forces. If we rewrite Eq. (2) in the similar variables of forced convection with q_{cT} = const, then this term acquires the form $2\sqrt{2}\,\mathrm{Gr}^*/\mathrm{Re}^{2.5}\int_0^\infty F\Theta d\eta$ where $F=U/U_e$, $\Theta=-(0{,}5\mathrm{Re})^{0{,}5}\times(T-T_e)\,\lambda/q_w x$, $\eta=(0{,}5\mathrm{Re})^{0{,}5}y/x$. If we now write the flow continuity equation in the same variables, multiply it by Θ, combine it with the energy transfer equation, written in the same variables, and integrate the result across the layer from 0 to ∞, then after allowing for the boundary conditions we have the equation

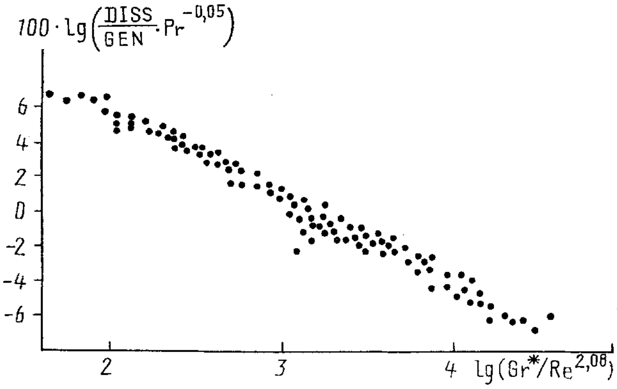

Fig. 2. Computed DISS/GEN points of the total energy flow.

$$2\xi \frac{d\,\mathrm{In}}{d\xi} + 2\,\mathrm{In} = 1/\mathrm{Pr},$$

where $\xi = \mathrm{Re}$; $\mathrm{In} \equiv \int_0^\infty F\Theta d\eta$. This equation has the analytical solution

$$\mathrm{In} = \frac{0,5}{\mathrm{Pr}}\left(1 - \frac{C}{\xi}\right). \tag{3}$$

From the computed results it was found that $C = \mathrm{Re}_0/\mathrm{Pr}^{0.1}$, where Re_0 depends on the starting point of the computation. Figure 1 shows the results of reducing the flow computations. Evaluated for $\mathrm{Re}_{init} = 4.75 \cdot 10^4$, $\mathrm{Pr} = 0.72$.

Generalizing what has been written above, we can write

$$\mathrm{GEN} = \frac{\beta g}{U_e^3}\int_0^\infty U(T - T_e)\,dy = -2,83\,\frac{\mathrm{Gr}^*}{\mathrm{Re}^{2,5}}\,\frac{0,5}{\mathrm{Pr}}\left(1 - \frac{\mathrm{Re}_0}{\mathrm{Pr}^{0,1}\,\mathrm{Re}}\right). \tag{4}$$

To explain the physical meaning of the constant Re_0 we examine Eq. (3) in more detail. First, the value of In does not depend on Gr^*. Hence it follows that by adding the natural flow to the forced flow the mutual variations of profiles of velocity and temperature are compensated for in such a way that In remains constant for a given value of Re.

Further, as test computations have shown, in the case of laminar forced flow In = 0.5/Pr i.e., C=0. This same conclusion can be derived from the similarity solution of the laminar forced convection problem with q_{cT} = const in these variables.

As was noted above, for the turbulent flow regime $C = \mathrm{Re}_0/\mathrm{Pr}^{0.1}$, where $\mathrm{Re}_0 \neq 0$. Then, from the fact that In must be a continuous function of Re it follows that C must vary with the development of the process. In the author's opinion, $C = C[(u'/U_e)_{max}]$, i.e., C = 0 for laminar flow, increases at the time of transition, and stabilizes in developed turbulence. Hence it follows that C, and this means also Re_0, characterize the point of transition to developed turbulence.

We can write Eq. (2) in the form

$$\mathrm{II} = \mathrm{III} - (\mathrm{I} + \mathrm{IV} + \mathrm{V} + \mathrm{VI}).$$

[Here we have replaced the terms by their ordinal number in Eq. (2)]. On the left is the variation of the turbulence energy, and on the right there is an increase of the total energy minus the decrease of total flow energy. Or we have

$$\mathrm{II} = \mathrm{III}\left(1 - \frac{\mathrm{I} + \mathrm{IV} + \mathrm{V} + \mathrm{VI}}{\mathrm{III}}\right). \tag{5}$$

The expression (I + IV + V + VI)/III describes the ratio of the dissipative terms to the production terms.

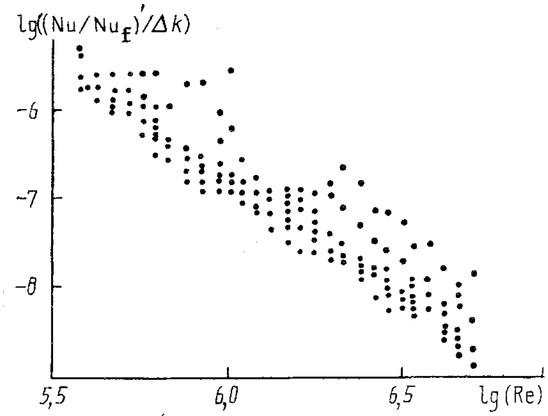

Fig. 3. Computed points $d(\mathrm{Nu}/\mathrm{Nu_f})/d\mathrm{Re}/)$ Δk.

Figure 2 shows the results of 25 computations of the boundary layer with different values of Pr and $\mathrm{Gr^*/Re^4}$. These results are well correlated by the relation

$$\frac{\mathrm{DISS}}{\mathrm{GEN}} = 1{,}5\,\mathrm{Pr}^{0,05}\left(\frac{\mathrm{Gr}^*}{\mathrm{Re}^{2,08}}\right)^{-0,0552}. \tag{6}$$

Figure 2 shows the computed point $\mathrm{DISS/GEN\cdot Pr^{-0.05}}$. Combining Eqs. (4)-(6) we have a correlation describing the decrease of turbulent energy, which means also laminarization of the flow when adding the natural convection to the forced convection:

$$\Delta k = \frac{d}{dx}\int_0^\infty\left[\left(\frac{kU}{U_e^3}\right)_{\mathrm{m}} - \left(\frac{kU}{U_e^3}\right)_{\mathrm{f}}\right]dy = 2{,}83\,\frac{\mathrm{Gr}^*}{\mathrm{Re}^{2,5}}\,\frac{0{,}5}{\mathrm{Pr}}\left(1 - \frac{\mathrm{Re_0}}{\mathrm{Pr}^{0,1}\,\mathrm{Re}}\right)\left(1 - 1{,}5\,\mathrm{Pr}^{0,05}\left(\frac{\mathrm{Gr}^*}{\mathrm{Re}^{2,08}}\right)^{-0,0552}\right). \tag{7}$$

It is known that the ratio $\mathrm{Nu/Nu_f}$, when adding natural convection to forced convection, decreases due to laminarization of the flow. Therefore, postulating changes of $\mathrm{Nu/Nu_f}$ and Δk in proportion, we consider the ratio $d(\mathrm{Nu/Nu_f})/dx/\Delta k$.

Figure 3 shows results of reducing the computed data. They are described satisfactorily by the correlation

$$\left.\frac{d(\mathrm{Nu/Nu_f})}{dx}\right/\Delta k = 1{,}07\cdot 10^6\,\mathrm{Re}^{-2,1}. \tag{8}$$

Combining Eqs. (7) and (8), we have the formula

$$\frac{d(\mathrm{Nu/Nu_f})}{dx} = \frac{1{,}514\cdot 10^6}{\mathrm{Pr}}\,\frac{\mathrm{Gr}^*}{\mathrm{Re}^{4,6}}\left(1 - \frac{\mathrm{Re_0}}{\mathrm{Pr}^{0,1}\,\mathrm{Re}}\right)\times\left(1 - 1{,}5\,\mathrm{Pr}^{0,05}\left(\frac{\mathrm{Gr}^*}{\mathrm{Re}^{2,08}}\right)^{-0,0552}\right). \tag{9}$$

We integrate this relation, putting that for $\mathrm{Re} = \mathrm{Re_0/Pr^{0.1}}$, $\mathrm{Nu} = \mathrm{Nu_f}$. As a result we have

$$\frac{\mathrm{Nu}}{\mathrm{Nu_f}} = 1 + \frac{10^6}{\mathrm{Pr}}\left[\frac{\mathrm{Gr}^*}{\mathrm{Re}^{3,6}}\left(3{,}29 + 2{,}2\,\frac{A}{\mathrm{Re}} - 5{,}49\left(\frac{A}{\mathrm{Re}}\right)^{0,4}\right) - \left(\frac{\mathrm{Gr}^*}{\mathrm{Re}^{3,69}}\right)^{0,945}\mathrm{Pr}^{0,05}\left(6{,}71 + 2{,}8\,\frac{A}{\mathrm{Re}} - 9{,}51\left(\frac{A}{\mathrm{Re}}\right)^{0,294}\right)\right],$$

where $A = \mathrm{Re_0/Pr^{0.1}}$.

Regarding its structure, this Eq. (10) has a transition to the formula for forced flow and describes a region of laminarized convection. It is valid, according to the test calculations, in the region $\mathrm{Nu/Nu_f} \le 1.5$ or $\mathrm{Gr^*/Re^{2.5}} \le 50$. To obtain the transition to formulas for heat transfer in natural convection we recommend the following expression:

TABLE 1. Comparison of Computations with the Experimental Data of [3] ($Pr = 0.72$, $Re_0 = 6\cdot10^4$)

$Re\times10^{-5}$	$Gr^*\times10^{-13}$	$Nu_{calc}\times10^{-2}$	$Nu_{exp}\times10^{-2}$	Deviation, %	$Re\times10^{-5}$	$Gr^*\times10^{-13}$	$Nu_{calc}\times10^{-2}$	$Nu_{exp}\times10^{-2}$	Deviation, %
3,05	3,63	5,33	5,90	—10,1	1,59	1,46	2,98	2,83	5,4
3,45	5,60	5,82	6,19	—5,9	1,71	1,93	3,09	2,98	3,6
2,10	9,30	4,11	4,53	—9,3	1,83	2,37	3,22	2,96	8,6
2,69	1,30	5,11	4,69	8,9	1,60	1,55	2,98	2,71	9,9
2,48	1,78	4,59	5,06	—9,3	1,72	2,05	3,08	2,89	6,6
2,84	2,95	5,04	5,49	—8,2	1,84	2,52	3,21	2,90	10,5
3,22	4,56	5,52	5,78	—4,6	2,08	3,93	3,42	3,09	10,7
2,11	1,44	3,97	4,04	—1,8	1,41	1,48	2,68	2,49	7,5
2,28	2,11	4,13	4,61	—10,5	1,52	2,00	2,76	2,76	—0,2
2,45	2,62	4,35	4,56	—4,7	1,62	2,54	2,83	2,90	—2,4
2,62	3,47	4,52	4,89	—7,5	1,84	4,10	2,99	3,36	—10,9
2,96	5,33	4,92	5,08	—3,2	9,80	1,28	2,18	2,41	—9,7
1,47	1,08	2,87	2,66	8,0					

TABLE 2. Comparison of the Computations with the Experimental Data of [4] ($Pr = 5.7$, $Re_0 = 7.5\cdot10^4$)

$Re\cdot10^{-5}$	$Gr^*\cdot10^{-14}$	$Nu_{calc}\,10^{-3}$	$Nu_{exp}\cdot10^{-3}$	Deviation, %
0,73	9,98	1,66	1,57	5,8
1,20	9,89	1,66	1,47	12,8
2,48	9,75	1,32	1,26	5,0
3,22	10,1	1,72	1,55	11,0
1,20	2,26	0,88	0,91	—3,9

$$\frac{Nu}{Nu_f}=\begin{cases}\text{Eq. (10)}, & Nu/Nu_f\leqslant 1{,}5,\\ Nu_{nat.}/Nu_f, & Nu/Nu_f>1{,}5.\end{cases}\qquad(11)$$

Here $Nu_{nat(ural)}$ is defined by any formula for computing heat transfer in natural convection.

Tables 1 and 2 compare Nu/Nu_f computed from Eq. (11) with the experimental data of [3] and [4] for air and water. The Nunat number was found from the Vliet–Laio formula: $Nu_{nat.}= 0.568\,(Gr^*Pr)^{0.22}$. The agreement is satisfactory. The Re_0 parameter was a free choice. In the comparison with [3] it is less, corresponding to the presence of a turbulence generator in the experiments.

Regarding determination of the value of Re_0 in the practical use of Eq. (11) the following comment can be made. As check computations have shown, a variation of Re_0 from $5\cdot10^4$ to 10^5 leads to no more than a 20% deviation of Nu/Nu_f, computed from Eq. (11), from the mean value of Nu/Nu_f in this interval of variation of Re_0, and this mean value coincides practically exactly with the value of Nu/Nu_f computed for $Re_0 = 7\cdot10^4$. However, in comparing the computations with the results of [4], obtained without a turbulence generator, the Re_0 value obtained was $7.5\cdot10^4$. For these reasons for the practical use of Eq. (11) we can recommend $Re_0 = 7\cdot10^4$.

CONCLUSIONS

The author has suggested a qualitative explanation of the occurrence of the phenomenon of laminarization when a secondary natural convection is added to a forced convection. To describe this process we propose the total flow energy balance equation in the turbulent mixed convection boundary layer. From analysis of solutions of this equation we propose correlations describing the development of laminarization of the outflow, and we have obtained a formula for heat transfer at the wall.

NOTATION

$k=0{,}5\,\overline{u_i' u_i'}$, turbulence energy; ε, rate of dissipation of turbulence energy; U, mean velocity in the longitudinal direction; U_e, incident flow velocity; T, mean temperature; T_e, temperature of the unperturbed flow; $\delta_e=\int_0^\infty U/U_e(1-(U/U_e)^2)dy$, energy displacement thickness; $Re = U_e x/\nu$, Reynolds number; $Nu = \alpha x/\lambda$, Nusselt number; Nu_f, Nusselt number for forced flow; $Gr^*= g\beta q_w x^4/\lambda\nu^2$, modified Grashof number; $Pr = \nu/a$, Prandtl number; q_{CT}, heat flux at the wall.

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